Exact Schedulability Analysis of Global Fixed Priority Scheduling by Using Linear Hybrid Automata

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Outline

1 Introduction

2 LHA models for multiprocessor G-FP scheduling

3 Weak Simulation Relation
   - Concrete state space
   - Symbolic state space

4 Experiments
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4. Experiments
Problem description

- $m$ processors
- $n$ sporadic tasks
- Global (task level) Fixed-Priority (G-FP) Preemptive Scheduling
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- $m$ processors
- $n$ sporadic tasks
- Global (task level) Fixed-Priority (G-FP) Preemptive Scheduling
- Is the taskset schedulable?
A sporadic task $\tau_i$ is specified by a tuple $(C, D, T)$
- $C$ is the worst-case execution time
- $D$ is the relative deadline
- $T$ is the minimum time interval between two successive job releases of the task
- $T \sim D$ with $\sim \in \{<, =, >\}$

Fully Preemptive

Tasks can migrate among different processors
Critical instant for G-FP scheduling?
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- Unknown
Critical instant for G-FP scheduling?

- Unknown
- An example (Baruah, 2007): \( \tau_1 = (1, 2, 2), \tau_2 = (1, 3, 3), \) and \( \tau_3 = (5, 6, 6) \)
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- Unknown
- An example (Baruah, 2007): $\tau_1 = (1, 2, 2)$, $\tau_2 = (1, 3, 3)$, and $\tau_3 = (5, 6, 6)$
State-of-the-art

Analytical (over-approximate) solutions: RTA-CE (Sun et al., 2014), RTA-LC (Guan et al., 2009)
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Our contributions

- A Linear Hybrid Automaton (LHA) model for exact G-FP scheduling
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- A **weak simulation relation** to simplify the state space exploration
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- A Linear Hybrid Automaton (LHA) model for exact G-FP scheduling
- A weak simulation relation to simplify the state space exploration
- An evaluation on the pessimism of the state-of-the-art analytical G-FP schedulability analysis
A Linear Hybrid Automaton is a tuple

\[ H = \{ V, D, L, \text{init}, \text{Lab}, T, \text{Invar} \} \]
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\[ H = \{ V, D, L, init, Lab, T, Invar \} \]

1. A finite set \( V = \{ x_1, \ldots, x_n \} \) of continuous variables.
2. A labeling function \( D \) which linearly constrains variables’ rate \( (\dot{V} = \{ \dot{x}_1, \ldots, \dot{x}_N \}) \) in each location.
Linear Hybrid Automata (LHA)

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2. A labeling function \( D \) which linearly constrains variables’ rate (\( \dot{V} = \{ \dot{x}_1, \ldots, \dot{x}_N \} \)) in each location.
3. A finite set \( L \) of locations.
4. An initial function \( init \).
5. A finite set \( Lab \) of synchronisation labels.
6. A finite set \( T \) of transitions (every location has an outgoing stutter transition to itself).
7. A labeling function \( Invar \) which assigns each location \( l \) an invariant.
A concrete state $s = (l, \nu)$: $l$ is a location and $\nu$ is a valuation over $\mathcal{V}$.
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- A transition $s_1 \to s_2$; a sequence of transitions $s_1 \Rightarrow s_2$
- The concrete state space of LHA: $\text{space}$
Concrete states and Symbolic States

- **A concrete state** $s = (l, \nu)$: $l$ is a location and $\nu$ is a valuation over $V$
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- **A symbolic state** $S = (l, C)$: $l$ is a location and $C$ is a linear constraint and can be represented by a convex region
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Examples

\[ s = (l, \nu) \text{ with } \nu = (1.6, 2.3) \]
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\[ s = (l, \nu) \text{ with } \nu = (1.6, 2.3) \]

\[ S = (l, C) \text{ with } C = \{1 \leq x \leq 3 \land x + y \leq 4\} \]
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A task \((C, D, T)\) is modeled by a LHA called Task Automaton (TA).

- Two continuous variables:
  1. \(p\) : the time passed since the latest task activation
  2. \(c\) : remaining computation time that needs to be executed
Task automaton

Idle
\[ \dot{p} = 1, \dot{c} = 0 \]
Task automaton

Idle
\[ \dot{p} = 1, \dot{c} = 0 \]

Waiting
\[ \dot{p} = 1, \dot{c} = 0 \]
\[ p \leq D \]

arrival
\[ p \geq T \]
\[ p := 0 \]
\[ c := C \]
Task automaton

Idle
\[ \dot{p} = 1, \dot{c} = 0 \]

Waiting
\[ \dot{p} = 1, \dot{c} = 0 \]
\[ p \leq D \]

Running
\[ \dot{p} = 1, \dot{c} = -1 \]
\[ c \geq 0 \land p \leq D \]

arrival
\[ p \geq T \]
\[ p := 0 \]
\[ c := C \]

dispatch
**Task automaton**

- **Idle**
  - \( \dot{p} = 1, \dot{c} = 0 \)

- **Waiting**
  - \( \dot{p} = 1, \dot{c} = 0 \)
  - \( p \leq D \)

- **Running**
  - \( \dot{p} = 1, \dot{c} = -1 \)
  - \( c \geq 0 \land p \leq D \)

- **Arrival**
  - \( p \geq T \)
  - \( p := 0 \)
  - \( c := C \)

- **Preemption**
  - \( c > 0 \)

- **Dispatch**
Task automaton

Idle
\[ \dot{p} = 1, \dot{c} = 0 \]

Waiting
\[ \dot{p} = 1, \dot{c} = 0 \]
\[ p \leq D \]

Running
\[ \dot{p} = 1, \dot{c} = -1 \]
\[ c \geq 0 \land p \leq D \]

arrival
\[ p \geq T \]
\[ p := 0 \]
\[ c := C \]

preemption
\[ c > 0 \]

dispatch

end
\[ c = 0 \]
Task automaton

Idle
\[ \dot{p} = 1, \dot{c} = 0 \]

Waiting
\[ \dot{p} = 1, \dot{c} = 0 \]
\[ p \leq D \]
\[ p \geq T \]
\[ p := 0 \]
\[ c := C \]

Running
\[ \dot{p} = 1, \dot{c} = -1 \]
\[ c \geq 0 \land p \leq D \]
\[ p \geq T \]
\[ p := 0 \]
\[ c := c + C \]

Arrival
\[ p \geq T \]
\[ p := 0 \]
\[ c := C \]

Preemption
\[ c > 0 \]

End
\[ c = 0 \]

Dispatch
\[ p \geq T \]
\[ p := 0 \]
\[ c := c + C \]
Task automaton

Idle
\[ p = 1, c = 0 \]

Waiting
\[ p = 1, c = 0 \]
\[ p \leq D \]

Running
\[ p = 1, c = -1 \]
\[ c \geq 0 \land p \leq D \]

Deadline Missed

arrival
\[ p \geq T \]
\[ p := 0 \]
\[ c := C \]

end
\[ c = 0 \]

preemption
\[ c > 0 \]

dispatch
\[ p \geq D \]

Deadline Missed

\[ p \geq T \]
\[ p := 0 \]
\[ c := c + C \]
The Scheduling Automaton (Sched)
- It synchronises with TAs and decides which tasks to run and which tasks to wait.
- It is a G-FP preemptive scheduler.

The System Automaton (SA)
- Composition of TAs and Sched: \( Sched \times TA_1 \times \cdots \times TA_n \)
The Scheduling Automaton (Sched)
- It synchronises with TAs and decides which tasks to run and which tasks to wait.
- It is a G-FP preemptive scheduler.

The System Automaton (SA)
- Composition of TAs and Sched: $Sched \times TA_1 \times \cdots \times TA_n$
- The schedulability problem is now the reachability problem of DeadlineMissed in SA.
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A weak simulation relation in concrete state space of SA is a preorder \( \preceq \subseteq \) space \( \times \) space such that:

1. \( \forall s_1, s_2, s_4 \text{ s.t. } s_1 \preceq s_2, s_2 \rightarrow s_4 \text{ there exists } s_3 \text{ s.t. } s_1 \Rightarrow s_3 \text{ and } s_3 \preceq s_4. \)

2. \( \forall s_1, s_2 \text{ s.t. } s_1 \preceq s_2 : \)

\( \text{s}_2 \text{ in DeadlineMissed implies } s_1 \text{ in DeadlineMissed} \)

Whenever \( s_1 \preceq s_2 \), we say that \( s_1 \) (weak) simulates \( s_2 \).
The slack-time pre-order relation $\succeq_{st}$

**Definition**

For the SA with a G-FP preemptive scheduler, its slack-time pre-order relation $\succeq_{st} \subseteq \text{space} \times \text{space}$ is defined such that $\forall s_1, s_2, s_1 \succeq_{st} s_2$ iff

$$\forall \tau_i : s_1 \cdot p_i \geq s_2 \cdot p_i \land s_1 \cdot c_i \geq s_2 \cdot c_i$$
The slack-time pre-order relation $\succeq_{st}$

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For the SA with a G-FP preemptive scheduler, its slack-time pre-order relation $\succeq_{st} \subseteq \text{space} \times \text{space}$ is defined such that $\forall s_1, s_2, s_1 \succeq_{st} s_2$ iff

$$\forall \tau_i : \ s_1.p_i \geq s_2.p_i \ \land \ s_1.c_i \geq s_2.c_i$$

**Theorem**

$\succeq_{st}$ is indeed a weak simulation relation in SA.
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A symbolic state $S = (I, C)$ abstracts a set of concrete states.

For two symbolic states $S_1$ and $S_2$, we say $S_1$ simulates $S_2$ if

$$\forall s_2 \in S_2, \ \exists s_1 \in S_1 \ \text{s.t.} \ \ s_1 \succeq s_2$$
A preliminary concept: convex region domination

- Assume a N-dimensional space
- Given two valuations $\nu = (\nu_1, \ldots, \nu_N)$ and $\nu' = (\nu'_1, \ldots, \nu'_N)$, we say $\nu$ dominates $\nu'$, denoted as $\nu \geq \nu'$, if $\forall i \in [1, N], \nu_i \geq \nu'_i$. 

A preliminary concept: convex region domination

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- Given two convex regions $C_1$ and $C_2$, we say $C_1$ dominates $C_2$, denoted as $C_1 \geq C_2$, if for any valuation $\nu'$ in $C_2$, there exists a valuation $\nu \in C_1$ such that $\nu \geq \nu'$. 
Definition

For the SA with a G-FP preemptive scheduler, its slack-time pre-order relation $\preceq_{st} \subseteq \text{Space} \times \text{Space}$ is defined such that $\forall S_1, S_2, S_1 \preceq_{st} S_2$ iff $S_1.C$ dominates $S_2.C$. 

Definition

For the SA with a G-FP preemptive scheduler, its slack-time pre-order relation \( \preceq_{st} \subseteq \text{Space} \times \text{Space} \) is defined such that \( \forall S_1, S_2, S_1 \preceq_{st} S_2 \) iff \( S_1.C \) dominates \( S_2.C \).

Theorem

\( \preceq_{st} \subseteq \text{Space} \times \text{Space} \) is a weak simulation relation.
How to decide $C_1 \geq C_2$?
Given a convex region $\mathcal{C}$

It's widening region $\nabla(\mathcal{C})$ is constructed as follows:
1. Construct linear constraints $\mathcal{C}'$ in $2 \times N$ dimensional space $(x_1, \ldots, x_N, y_1, \ldots, y_N)$ such that

\[(y_1, \ldots, y_N) \models \mathcal{C} \quad \land \quad \forall i, \ x_i \leq y_i\]

2. Remove the space dimensions higher than $N$ in $\mathcal{C}'$. 

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An example of widening

\[ x + y \leq 4 \]
\[ x \geq 1 \]
\[ y \geq 1 \]

Diagram:

- Region shaded in blue: \( C \)
- Axes: \( x \) and \( y \)
- Grid lines
- Labels: \( x \geq 1 \) and \( y \geq 1 \)
An example of widening:

\[ x \geq 1 \quad y \geq 1 \quad x + y \leq 4 \]

\[ y \leq 3 \quad x \leq 3 \]

\( C \)

\( \nabla(C) \)
Lemma

Given two convex regions $C_1$ and $C_2$, $C_1 \geq C_2$ if and only if $\nabla(C_1)$ includes $\nabla(C_2)$.

Proof.
Algorithm 1 Schedulability Analysis in SA (SA-SA)

1: $R \leftarrow \{S_0\}$
2: while true do
3: \hspace{1em} $P \leftarrow \text{Post}(R)$
4: \hspace{1em} if $P \cap F \neq \emptyset$ then
5: \hspace{2em} return NOT schedulable
6: \hspace{1em} end if
7: \hspace{1em} $R' \leftarrow R \cup P$
8: \hspace{1em} $R' \leftarrow \text{Max}\subseteq(R')$
9: \hspace{1em} if $R' = R$ then
10: \hspace{2em} return schedulable
11: \hspace{1em} else
12: \hspace{2em} $R \leftarrow R'$
13: \hspace{1em} end if
14: end while
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RTA-CE: Response Time Analysis with Carry-in Enumeration
(Sun et al., 2014)
RTA-CE : Response Time Analysis with Carry-in Enumeration (Sun et al., 2014)

- $m \in \{2, 3\}$ and $n = 5$
- $T_i \in [100, 1000]$ and a series of taskset utilisation levels $U$ separated by 0.1
- For each $(m, n, U)$ configuration 100 tasksets are generated by Randomfixedsum (Emberson et al., 2010)
Results I: $m = 2$, $n = 5$, $\frac{D_i}{T_i} \in [0.8, 1]$
Results II: $m = 2$, $n = 5$, $\frac{D_i}{T_j} \in [0.8, 1.2]$
Results IV: $m = 3$, $n = 5$, $\frac{D_i}{T_j} \in [0.8, 1]$
SA-SA vs. SA-SA-WOS

- SA-SA-WOS: SA-SA WithOut Simulation
- \( m = 2, \ n = 5, \ \frac{D_i}{T_i} \in [0.8, 1], \) and \( U \in [1, 1.6] \)
- 100 task sets
Results
SA-SA: another experiment on complexity

- $m = 2$, $n = 6$, $\frac{D_i}{T_i} \in [0.8, 2]$, and $U \in [1, 2]$
- 50 task sets
Results

Time (minutes) vs. NO. of states graph showing a trend that time increases with the number of states.
Questions?