Static Probabilistic Timing Analysis of Random Replacement Caches using Lossy Compression

David Griffin, Benjamin Lesage, Alan Burns, Rob Davis
Introduction

- Static analysis gives absolute guarantees
- ... but it's massively pessimistic
- ... and most people don't need absolute guarantees
Static Probabilistic Timing Analysis (SPTA)

- Determine probability that a system would fail
- Find a probability of failure that is sufficiently low
- (Hopefully less pessimistic)
Randomised Hardware

- Can be analysed by SPTA
- Idea: By making a lot of truly random choices, expected behaviour is predictable
- This paper examines the Random Replacement cache
Random Replacement Cache

- If memory access is a hit, do nothing

```
| a | b | c | d |
```

access c

```
| a | b | c | d |
```
Random Replacement Cache

- If memory access is a miss, evict a random element

```
| a | b | c | d |
```

access e

```
| p(0.25) e | b | c | d |
```
```
| p(0.25) a | e | c | d |
```
```
| p(0.25) a | b | e | d |
```
```
| p(0.25) a | b | c | e |
```
Current Analysis

- Original methods assumed independence of hit probabilities
  - They're not
- Corrected version by Davis et al. [5]
  - But this simple analysis is outperformed by LRU cache in all cases [Reineke, 2014]
State of the Art Techniques

- Focus Blocks approach proposed by Altmeyer and Davis [6]
- A number of memory blocks are focused and analysed by exhaustive search
- Others are analysed by previous method
- Not dominated by LRU analysis
State of the Art Techniques

- Weaknesses of Focus Blocks
  - Computationally expensive
  - Increasing number of focus blocks can decrease accuracy
  - Size of distributions modelled increases with size of input
  - No May analysis
Lossy Compression

- Lossy Compression is used to find what can be removed with little consequence.
- In this case, apply lossy compression to states in exhaustive search.
## Exhaustive Search Example

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>b</th>
<th>a</th>
<th>b</th>
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<tbody>
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<td>[x, x]</td>
<td>p=1</td>
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<td>[a, c]</td>
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<td>h(0)=1</td>
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<td>h(0)=1/4</td>
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<td>h(0)=1/16</td>
</tr>
</tbody>
</table>
Exhaustive Search Example

\[
\begin{array}{|c|c|c|c|c|}
\hline
a & b & c & b & a \\
\hline
[x, x] & [a, x] & [a, b] & [a, c] & [a, b] \\
p=1 & p=1 & p=1/2 & p=1/4 & p=6/16 \\
h(0)=1 & h(0)=1 & h(0)=1/2 & h(0)=1/4 & h(0)=2/16 \\
\hline
[b, x] & & [b, c] & & [b, c] \\
p=1/2 & & p=2/4 & & p=6/8 \\
h(0)=1/2 & & h(0)=2/4 & & h(0)=2/8 \\
\hline
[c, x] & & [c, x] & & [b, x] \\
p=1/4 & & p=1/8 & & p=9/16 \\
h(0)=1/4 & & h(0)=1/8 & & h(0)=3/16 \\
\hline
\end{array}
\]
### Exhaustive Search Example

<table>
<thead>
<tr>
<th>Column</th>
<th>Row 1</th>
<th>Row 2</th>
<th>Row 3</th>
<th>Row 4</th>
<th>Row 5</th>
</tr>
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<tbody>
<tr>
<td>a</td>
<td>[x, x]</td>
<td>p=1</td>
<td>h(0)=1</td>
<td>[a, x]</td>
<td>p=1</td>
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<tr>
<td>b</td>
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<tr>
<td>b</td>
<td>[a, c]</td>
<td>p=1/4</td>
<td>h(0)=1/4</td>
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<td>[b, c]</td>
</tr>
<tr>
<td>a</td>
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<tbody>
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<td><img src="image.png" alt="Diagram" /></td>
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</tbody>
</table>
Exhaustive Search Example

\[
\begin{align*}
[x, x] & : p = 1 \\
h(0) = 1 \\
\end{align*}
\]

\[
\begin{align*}
[a, x] & : p = 1 \\
h(0) = 1 \\
\end{align*}
\]

\[
\begin{align*}
[a, b] & : p = 1/2 \\
h(0) = 1/2 \\
[b, x] & : p = 1/2 \\
h(0) = 1/2 \\
\end{align*}
\]

\[
\begin{align*}
[a, c] & : p = 1/4 \\
h(0) = 1/4 \\
[b, c] & : p = 2/4 \\
h(0) = 2/4 \\
[c, x] & : p = 1/4 \\
h(0) = 1/4 \\
\end{align*}
\]

\[
\begin{align*}
[a, b] & : p = 1/8 \\
h(0) = 1/8 \\
[b, c] & : p = 6/8 \\
h(0) = 2/8 \\
h(1) = 4/8 \\
[c, x] & : p = 1/8 \\
h(0) = 1/8 \\
\end{align*}
\]

\[
\begin{align*}
[a, b] & : p = 6/16 \\
h(0) = 2/16 \\
h(1) = 4/16 \\
[b, c] & : p = 9/16 \\
h(0) = 3/16 \\
h(1) = 6/16 \\
[a, x] & : p = 1/16 \\
h(0) = 1/16 \\
\end{align*}
\]
Exhaustive Search Example

- [x, x]
p = 1
h(0) = 1
- [a, x]
p = 1
h(0) = 1
- [a, b]
p = 1/2
h(0) = 1/2
- [b, x]
p = 1/2
h(0) = 1/2
- [a, c]
p = 1/4
h(0) = 1/4
- [b, c]
p = 2/4
h(0) = 1/2
- [c, x]
p = 1/4
h(0) = 1/4
- [a, b]
p = 1/8
h(0) = 1/8
- [b, c]
p = 6/8
h(0) = 2/4
- [c, x]
p = 1/8
h(0) = 1/4
- [a, b]
p = 6/16
h(0) = 2/16
- [b, c]
p = 9/16
h(0) = 3/16
- [c, x]
p = 1/16
h(0) = 1/16
- [a, b]
p = 4/16
h(0) = 4/16
- [b, c]
p = 16/16
h(0) = 8/16
- [c, x]
p = 1/16
h(0) = 1/16
Exhaustive Search Example

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Exhaustive Search
Example

\[
\begin{array}{c|c|c|c|c|c|c|c}
\hline
& a & b & c & b & a \\
\hline
[x, x] & [a, x] & [a, b] & [a, c] & [a, b] & [a, c] \\
h(0)=1 & h(0)=1 & h(0)=1/2 & h(0)=1/4 & h(0)=1/8 & h(0)=2/16 \\
\hline
[b, x] & [b, c] & [b, x] & [b, c] & [b, c] & [a, b] \\
h(0)=1/2 & h(0)=2/4 & h(0)=1/4 & h(0)=2/8 & h(0)=3/16 & h(0)=6/16 \\
\hline
\end{array}
\]
Exhaustive Search
Example

\[
\begin{array}{c|c|c|c|c|}
\text{a} & \text{b} & \text{c} & \text{a} & \text{b} \\
\hline
[x, x] & [a, x] & [a, b] & [a, c] & [a, b] \\
p=1 & p=1 & p=1/2 & p=1/4 & p=1/8 \\
h(0)=1 & h(0)=1 & h(0)=1/2 & h(0)=1/4 & h(0)=1/8 \\
\end{array}
\]

- [b, x]: p=1/2, h(0)=1/2
- [b, c]: p=2/4, h(0)=1/2
- [c, x]: p=1/4, h(0)=1/2
- [b, x]: p=1/8, h(0)=1/4
- [b, c]: p=6/8, h(0)=1/4
- [c, x]: p=6/8, h(0)=1/4
- [b, x]: p=9/16, h(0)=1/4
- [b, c]: p=1/8, h(0)=1/4
- [a, x]: p=1/16, h(0)=1/4

- [b, c] + [c, x] = [a, c] = p=6/16, h(0)=2/16, h(1)=4/16
What to compress

- Problem: All information is potentially valuable
- Solution: Decide based on context when information is not likely to be valuable
Compressing Memory Blocks

- Want to discard least important memory blocks
- Assume that importance correlates to hit probability
- Define $\emptyset$ to be an unknown memory block
Compressing Memory Blocks

- If a memory block isn't used again in sufficient time, it can be inferred it's hit probability will become low, and therefore it isn't important.
  - FRD(x): Replace any memory block whose forward reuse distance is > x with ∅
Example - FRD(2)

\[
\begin{align*}
[a] & : \emptyset, \emptyset \\
& \quad p = 1 \\
& \quad h(0) = 1 \\
[b] & : \emptyset, \emptyset \\
& \quad p = 1 \\
& \quad h(0) = 1 \\
[c] & : b, \emptyset \\
& \quad p = \frac{1}{2} \\
& \quad h(0) = \frac{1}{2} \\
& \quad h(1) = \frac{1}{2} \\
[d] & : \emptyset, \emptyset \\
& \quad p = 1 \\
& \quad h(0) = \frac{1}{2} \\
& \quad h(1) = \frac{1}{2} \\
\end{align*}
\]
Example - FRD(2)

[∅, ∅]  
\[p=1\]  
\[h(0)=1\]

[∅, ∅]  
\[p=1\]  
\[h(0)=1\]

[b, ∅]  
\[p=1/2\]  
\[h(0)=1/2\]

[b, ∅]  
\[p=1\]  
\[h(0)=1/2\]  
\[h(1)=1/2\]

[∅, ∅]  
\[p=1\]  
\[h(0)=1/2\]  
\[h(1)=1/2\]
Example - FRD(2)

\[ \emptyset, \emptyset \]
\[ p=1 \]
\[ h(0)=1 \]

\[ \emptyset, \emptyset \]
\[ p=1 \]
\[ h(0)=1 \]

\[ b, \emptyset \]
\[ p=1 \]
\[ h(0)=1 \]

\[ \emptyset, \emptyset \]
\[ p=1/2 \]
\[ h(0)=1/2 \]
\[ h(1)=1/2 \]

\[ \emptyset, \emptyset \]
\[ p=1 \]
\[ h(0)=1/2 \]
\[ h(1)=1/2 \]
Example - FRD(2)

\[
\begin{align*}
\text{a) } & [\emptyset, \emptyset] \\
& p=1 \\
& h(0)=1 \\

\text{b) } & [\emptyset, \emptyset] \\
& p=1 \\
& h(0)=1 \\

\text{c) } & [b, \emptyset] \\
& p=1/2 \\
& h(0)=1/2 \\
& \quad [\emptyset, \emptyset] \\
& p=1/2 \\
& h(0)=1/2
\end{align*}
\]
Compressing Memory Blocks

- Or, simply calculate the hit probability of elements in cache
  - PRB(x): Replace any memory block with a hit probability of < x with ∅
  - Not as aggressive as FRD
<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
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<tbody>
<tr>
<td>x</td>
<td>[x, x]</td>
<td>[a, x]</td>
<td>[a, b]</td>
<td>[c, ∅]</td>
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<tr>
<td></td>
<td>m(0)=1</td>
<td>m(1)=1</td>
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<td>m(3)=1/4</td>
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<td>m(4)=2/4</td>
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</tbody>
</table>
Example – PRB(0.5)

\[
\begin{align*}
[x, x] & \quad p=1 \\
m(0)=1
\end{align*}
\]

\[
\begin{align*}
[a, x] & \quad p=1 \\
m(1)=1
\end{align*}
\]

\[
\begin{align*}
[a, b] & \quad p=1/2 \\
m(2)=1/2
\end{align*}
\]

\[
\begin{align*}
[c, \emptyset] & \quad p=1/2 \\
m(3)=1/2
\end{align*}
\]

\[
\begin{align*}
[b, \emptyset] & \quad p=1/4 \\
m(3)=1/4
\end{align*}
\]

\[
\begin{align*}
[a, \emptyset] & \quad p=1/8 \\
m(3)=1/8
\end{align*}
\]

\[
\begin{align*}
[b, x] & \quad p=1/2 \\
m(2)=1/2
\end{align*}
\]

\[
\begin{align*}
[b, c] & \quad p=3/4 \\
m(3)=1/4
\end{align*}
\]

\[
\begin{align*}
[b, c] & \quad p=3/8 \\
m(4)=1/8
\end{align*}
\]

\[
\begin{align*}
[a, c] & \quad p=4/8 \\
m(3)=1/8
\end{align*}
\]
Example – PRB(0.5)

[\{x, x\}  
  \text{p=1}  
  \text{m(0)=1}

[\{a, x\}  
  \text{p=1}  
  \text{m(1)=1}

[\{a, b\}  
  \text{p=1/2}  
  \text{m(2)=1/2}

[\{c, \emptyset\}  
  \text{p=1/2}  
  \text{m(3)=1/2}

[\{b, \emptyset\}  
  \text{p=1/4}  
  \text{m(3)=1/4}

[\{a, \emptyset\}  
  \text{p=1/8}  
  \text{m(3)=1/8}

[\{b, x\}  
  \text{p=1/2}  
  \text{m(2)=1/2}

[\{b, c\}  
  \text{p=1/2}  
  \text{m(3)=1/2}

[\{b, c\}  
  \text{p=3/4}  
  \text{m(3)=1/4}

[\{b, c\}  
  \text{p=3/8}  
  \text{m(4)=1/8}

[\{a, c\}  
  \text{p=3/8}  
  \text{m(4)=3/8}

[\{a, b\}  
  \text{p=4/8}  
  \text{m(3)=1/8}

[\{a, b\}  
  \text{p=4/8}  
  \text{m(3)=1/8}  
  \text{m(4)=1/8}  
  \text{m(5)=2/8}
Compressing Distributions

- We are interested in the tail of a distribution
- .. but not interested in the extreme tail of the distribution.
  - Compress any event in distribution that is sufficiently unlikely
    - e.g. probability < $10^{-9}$
Compressing Distributions

- Use fractions for probabilities \( \frac{a}{b} \)
- If \( b \) exceeds a threshold \( \alpha \), simplify by a fixed factor \( f \)

\[
\max \left( \left\{ \frac{x}{\left\lfloor \frac{b}{f} \right\rfloor} \left| \frac{x}{\left\lceil \frac{b}{f} \right\rceil} < \frac{a}{b} \right\} \right) \]

- Combine all probability 'lost' due to simplification and place into an upper bound state
Example $\alpha = 8, f = 2$

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
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<tr>
<td>$[x, x]$</td>
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<tr>
<td>$p=1$</td>
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<tr>
<td>$h(0)=1$</td>
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<td>$h(0)=1/8$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b</th>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[b, x]$</td>
<td>$[a, c]$</td>
</tr>
<tr>
<td>$p=1/2$</td>
<td>$p=1/4$</td>
</tr>
<tr>
<td>$h(0)=1/2$</td>
<td>$h(0)=1/4$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b</th>
<th>a</th>
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<tbody>
<tr>
<td>$[b, c]$</td>
<td>$[a, b]$</td>
</tr>
<tr>
<td>$p=2/4$</td>
<td>$p=1/8$</td>
</tr>
<tr>
<td>$h(0)=2/4$</td>
<td>$h(0)=1/8$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>b</th>
<th>a</th>
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<tbody>
<tr>
<td>$[c, x]$</td>
<td>$[a, c]$</td>
</tr>
<tr>
<td>$p=1/4$</td>
<td>$p=1/4$</td>
</tr>
<tr>
<td>$h(0)=1/4$</td>
<td>$h(0)=1/4$</td>
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</tbody>
</table>
**Example** \( a = 8, f = 4 \)

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>b</th>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[x, x]</td>
<td>[a, x]</td>
<td>[a, b]</td>
<td>[a, c]</td>
<td>[a, b]</td>
</tr>
<tr>
<td>p</td>
<td>p=1</td>
<td>p=1</td>
<td>p=1/2</td>
<td>p=1/4</td>
<td>p=1/8</td>
</tr>
<tr>
<td>h0</td>
<td>h(0)=1</td>
<td>h(0)=1</td>
<td>h(0)=1/2</td>
<td>h(0)=1/4</td>
<td>h(0)=1/8</td>
</tr>
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</tr>
<tr>
<td></td>
<td>[b, x]</td>
<td>[b, c]</td>
<td>[c, x]</td>
<td>[b, c]</td>
<td>[b, x]</td>
</tr>
<tr>
<td>p</td>
<td>p=1/2</td>
<td>p=2/4</td>
<td>p=1/4</td>
<td>p=6/8</td>
<td>p=1/8</td>
</tr>
<tr>
<td>h0</td>
<td>h(0)=1/2</td>
<td>h(0)=2/4</td>
<td>h(0)=1/4</td>
<td>h(0)=2/8</td>
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</tbody>
</table>

[The University of York]
**Example** \( a = 8, f = 2 \)

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>b</th>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[x, x]</td>
<td>[a, x]</td>
<td>[a, b]</td>
<td>[a, c]</td>
<td>[a, b]</td>
</tr>
<tr>
<td></td>
<td>( p = 1 )</td>
<td>( p = 1 )</td>
<td>( p = 1/2 )</td>
<td>( p = 1/4 )</td>
<td>( p = 1/8 )</td>
</tr>
<tr>
<td></td>
<td>( h(0) = 1 )</td>
<td>( h(0) = 1 )</td>
<td>( h(0) = 1/2 )</td>
<td>( h(0) = 1/4 )</td>
<td>( h(0) = 1/8 )</td>
</tr>
<tr>
<td></td>
<td>[b, x]</td>
<td>[b, c]</td>
<td>[b, c]</td>
<td>[b, x]</td>
<td>[a, ∅]</td>
</tr>
<tr>
<td></td>
<td>( p = 1/2 )</td>
<td>( p = 2/4 )</td>
<td>( p = 6/8 )</td>
<td>( p = 1/4 )</td>
<td>( p = 1/4 )</td>
</tr>
<tr>
<td></td>
<td>( h(0) = 1/2 )</td>
<td>( h(0) = 2/4 )</td>
<td>( h(0) = 2/8 )</td>
<td>( h(0) = 1/4 )</td>
<td>( h(0) = 1/4 )</td>
</tr>
<tr>
<td></td>
<td>[c, x]</td>
<td>[c, x]</td>
<td>[c, x]</td>
<td>[b, x]</td>
<td>[a, ∅]</td>
</tr>
<tr>
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<td>( p = 1/4 )</td>
<td>( p = 1/4 )</td>
<td>( p = 1/8 )</td>
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</table>
New Technique

- Two proposed techniques
  - FRD – Forward Reuse Distance
  - PRB – Hit Probability
- Combines the FRD/PRB memory block compression strategies with distribution compression
Evaluation

- 16-way Random Replacement Cache with cache line size 8
- Traces from Mälardalen Benchmarks
- Fixed parameters $\alpha = 10^9$, $t = 10^6$
- Variety of parameters for PRB and FRD methods
Evaluation

- Compared against
  - 1 Billion Simulator Runs
  - Altmeyer and Davis' Focus Blocks method [6]
- All analyses run on moderately powerful laptop
  - (Does not include simulator runs)
Evaluation – insertsort

hits
Evaluation – insertsort hits
Evaluation – insertsort hits

\begin{figure}
\centering
\includegraphics[width=\textwidth]{chart.png}
\caption{CDF of hits for PRB(0.5) and FRD(24)}
\end{figure}
Evaluation – insertsort hits
Evaluation – fir hits

![Graph showing 1-CDF versus Hits for simulation, FRD(40), FRD(20), FB(12), PRB(0.5), PRB(0.6), and rd]
Evaluation – fir hits

At high probability, compression techniques are outperformed.
Evaluation – fir hits

At high probability, compression techniques are outperformed.

To stay ahead at higher accuracy, would need greater α parameter.
Evaluation – bs hits
Evaluation – bs hits

Small difference of parameter can make a large difference
Evaluation – execution time for iterations of fibcall

![Graph showing execution time vs. number of iterations for different methods: FRD(12), FRD(20), PRB(0.5), FB(12), FB(8). The graph illustrates the time taken in seconds for different numbers of iterations.]
Evaluation – execution time for iterations of fibcall

All parameters have very similar accuracy, apart from FB(8)
Evaluation – execution time for iterations of fibcall

Lower complexity
Evaluation – insertsort misses
Evaluation – insertsort misses

FRD < 116 gives near zero Misses
Evaluation – insertsort
misses

PRB now outperforms FRD
Conclusions

- Parameters improve accuracy in a predictable way
- Significant improvements over previous methods in accuracy, speed and memory usage
- Demonstrates a May Analysis for a Random Replacement cache is possible
Future Work

- “Fixed Effort” compression
  - Current technique is analogous to “Fixed Quality” compression
  - “Fixed Effort” is analogous to “Fixed Bitrate”
  - Idea: Given finite time, calculate best possible result
- Extension to multipath
Any Questions?